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ALONGSHORE VARIATION IN BEACH CUSP MORPHOLOGY IN A COASTAL EMBAYMENT

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ABSTRACT

The variation in beach cusp characteristics was examined along a 1 km long embayed beach (Pearl Beach, New South Wales, Australia). The beach cusp morphology had formed during the previous day and/or night and displayed a marked alongshore variation in cusp spacing. The edge wave mechanism of beach cusp formation could not account for the observed trend in cusp spacing, because no relationship could be established between the spacing of the cusps and the gradient of the beachface. On the other hand, the cusp spacing was strongly related to the horizontal swash excursion, providing some support for the self-organization model of beach cusp formation. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: beach cusps; beach; swash zone; self-organization; edge waves

INTRODUCTION

Beach cusps are morphological features found on the beachface, consisting of steep-gradient, seaward-pointing cusp horns and gentle-gradient embayments. They are formed by swash action and usually develop under accretionary wave conditions (Takeda and Sunamura, 1983; Masselink *et al.,* 1997). Beach cusps often display a marked alongshore rhythmicity, referred to as cusp spacing.

It is well established that beach cusps develop and are maintained by a three-dimensional swash flow circulation pattern characterized by wave uprush being deflected from the horns into the adjacent embayments (Bagnold, 1940; Russell and McIntire, 1962; Dean and Maurmeyer, 1980; Werner and Fink, 1993; Masselink *et al.,* 1997). This swash circulation pattern is referred to as 'horn-divergent flow' (Masselink and Pattiaratchi, 1998) and promotes onshore sediment transport and steep gradieints on the cusp horns, and offshore transport and gentle gradients in the embayments. The mechanisms of initial formation of beach cusp morphology, however, remains an enigma, and at present two explanations for beach cusp formation exist: (1) standing edge waves; and (2) self-organization.

Edge wave theory considers standing edge waves to be responsible for the development of rhythmic cusp morphology (Guza and Inman, 1975; Guza and Bowen, 1981). According to this theory, swash from incident waves is superimposed on the motion of standing edge waves to produce a systematic alongshore variation in swash height, resulting in a regular erosional perturbation. Positive feedback processes between swash circulation and beachface morphology then enhance these topographic perturbations resulting in beach cusp morphology (Inman and Guza, 1982). The two edge wave types that are generally implicated in the formation of beach cusps are mode zero, sub-harmonic edge waves and synchronous edge waves (Guza and Inman, 1975). The edge wave theory predicts a cusp spacing λ of:

$$\lambda = \frac{g}{\pi} T^2 \tan\beta \tag{1}$$

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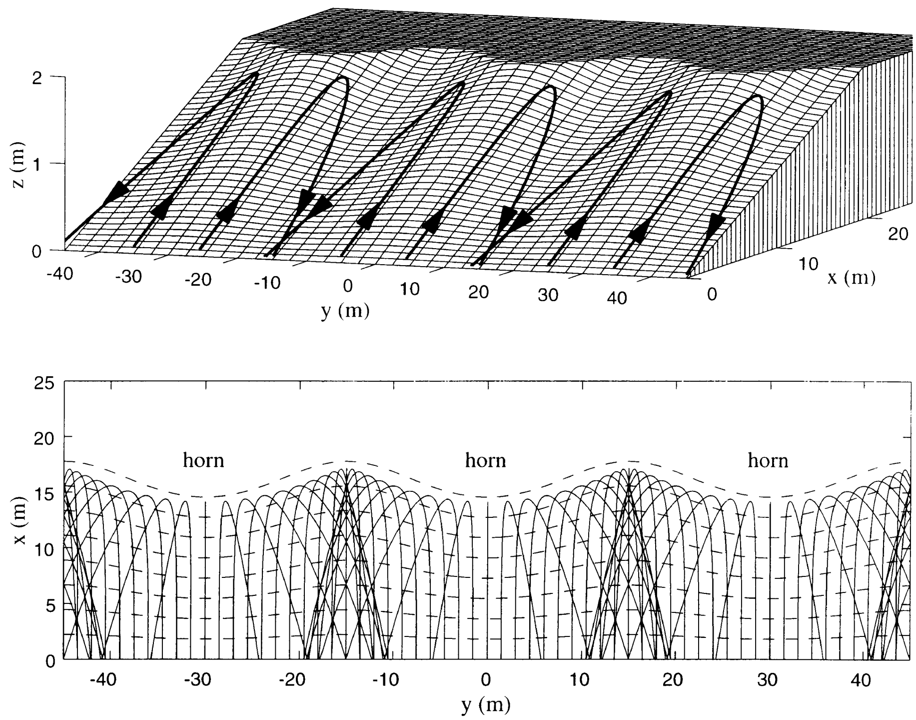

Figure 1. Motion of water particles under the influence of gravity and an initial upslope velocity on idealized cusp morphology according to the numerical model of Masselink and Pattiaratchi (1998). In the model, the water particles are allowed to pass through each other uninterrupted. In reality, the deflected uprush from the opposing flanks of the cusp horns meet in the centre of the embayments where they join to form concentrated backwash streams. The upper panel shows an oblique view of the beach cusp morphology and the motion of particles released at $y = -35, -20, -5, 5, 25$ and 35 m. The lower panel shows a contour plot of the cusp morphology (contour lines every 0·25 m) and the motion of particles released every 1·5 m along the beach cusp morphology (input parameters; $S = 15$ m, $\lambda = 30$ m, $\varepsilon = 0{\cdot}1$ and $\varepsilon(S/\lambda)^2 = 0{\cdot}025$)

for the sub-harmonic case and:

$$\lambda = \frac{g}{2\pi} T^2 \tan\beta \tag{2}$$

for the synchronous case, where T is the incident wave period and tan β is the beach gradient.

The self-organization theory of beach cusp formation considers beach cusps to be the result of feedback processes between morphology and swash flow (Werner and Fink, 1993). Positive feedback enhances random morphologic irregularities, whereas negative feedback inhibits erosion and accretion on well developed cusps. Morphological regularity arises from the internal dynamics of the system and hence the term 'self-organization' is employed. The dimension of the ryhthmic morphology is scaled by the horizontal swash excursion S according to:

$$\lambda = fS \tag{3}$$

where the value of the coefficient f lies between 1 and 3. Werner and Fink (1993) do not provide a physical explanation for the scaling of the cusp length by the swash excursion. However, using a simple model of water particle kinematics, Dean and Maurmeyer (1980) indicate that the three-dimensional

motion of water particles on idealized cusp morphology is largely a function of the ratio cusp spacing to swash excursion (i.e. f). The smaller f (large S and/or small λ), the larger the tendency of water particles running up the cusp morphology to be deflected into the embayments under the influence of lateral gradients associated with beach cusp morphology. According to Dean and Maurmeyer (1980), maximum deflection of the wave uprush into the embayment occurs for $f = 1{\cdot}5$.

Divergence of the swash flow at the locations of the cusp horns (and convergence of the flow into the cusp embayments) is also expected to depend on the prominence of the beach cusp morphology. On pronounced beach cusps, lateral beachface gradients are larger than on subdued beach cusps and consequently the degree of horn divergence will be greater. Masselink and Pattiaratchi (1998) developed a numerical model capable of simulating the motion of water particles on idealized beach cusp morphology (Figure 1) and demonstrate that the degree of horn-divergent flow depends on the parameter $\varepsilon(S/\lambda)^2$, where:

$$\varepsilon = \frac{\tan\beta_{horn} - \tan\beta_{bay}}{\tan\beta_{horn} + \tan\beta_{bay}} \tag{4}$$

and $\tan\beta_{horn}$ and $\tan\beta_{bay}$ are the beach gradient of the cusp horn and embayment, respectively (cf. Dean and Maurmeyer, 1980). The value of ε increases with the prominence of the cusp morphology. When $\varepsilon(S/\lambda)^2 < 0{\cdot}015$, the beach cusps are large and/or subdued in relation to the swash excursion. The swash circulation in such cases is essentially two-dimensional with uprush and backwash displaying a simple up-down motion. Deflection of the wave uprush from the horns into the embayments increases rapidly with $\varepsilon(S/\lambda)^2$. Maximum horn-divergent flow is attained for $\varepsilon(S/\lambda)^2 = 0{\cdot}15$ and under such conditions all the water running up the beachface will eventually leave the cusp system through the embayment. The non-dimensional parameter $\varepsilon(S/\lambda)^2$ can be rewritten as:

$$\lambda = K\sqrt{\varepsilon}S \tag{5}$$

where K is a dimensionless coefficient. Equation 5 cannot be employed to predict λ without prior knowledge of ε. However, the relation can be validated using well developed and equilibrium beach cusp morphology to gain insight into the dynamics of cusp maintenance.

There are two avenues of investigation available to validate the edge wave and self-organization theories of beach cusp formation. The first involves detailed monitoring of the water motion in the swash and surf zones during beach cusp formation. Such an investigation was carried out by Holland and Holman (1996) and they did not find any support for the edge wave mechanism of beach cusp generation. The second approach compares predicted (Equations 1–3) with observed beach cusp spacing and has been followed by numerous researchers (e.g. Huntley and Bowen; 1978; Sallenger, 1979; Dean and Maurmeyer, 1980; Inman and Guza, 1982; Sunamura and Takeda, 1983; Seymour and Aubrey, 1985). To date, the results of these studies have been inconclusive, primarily due to the uncertainty in defining the appropriate incident wave period and beach gradient (Holland and Holman, 1996), and the similarity between predicted cusp spacing according to both theories (Werner and Fink, 1993).

In a coastal embayment, morphological and hydrodynamic parameters such as beach gradient, sediment size, cusp spacing, wave height and swash excursion may vary along its shoreline (e.g. Krumbein, 1944). In contrast, the wave period remains relatively uniform along the embayment, in particular under narrow-banded swell conditions. If a single-generation set of beach cusps (formed at the same time under similar offshore forcing conditions) is present within such an environment, an ideal opportunity is offered to investigate the two models of beach cusp formation. If edge wave theory is valid, the alongshore variation in cusp spacing should be explained in terms of the beach gradient. If, however, the self-organization model is valid, the alongshore variation in cusp spacing should be accounted for by the alongshore variation in the horizontal swash excursion. The aim of the present paper is to compare the variation in beach cusp morphology measured along an embayed beach to that predicted by Equations 1–3 to validate the standing edge wave theory and the self-organization mechanism of beach cusp formation.

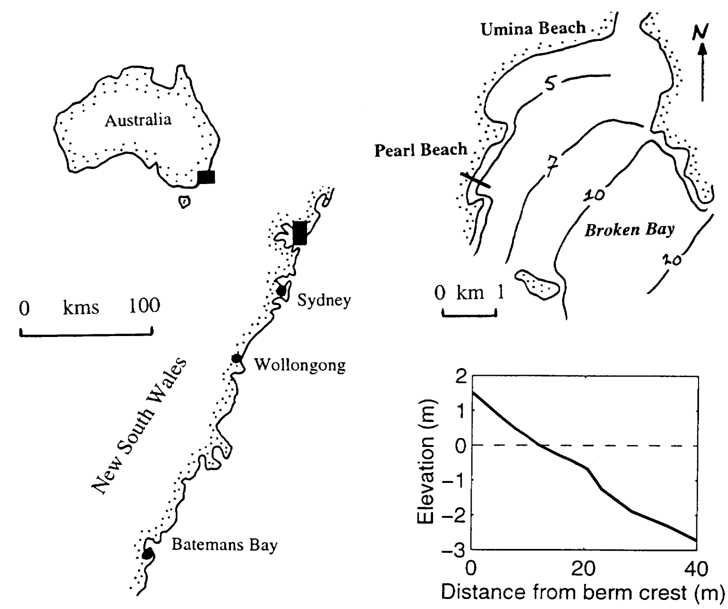

Figure 2. Location of Pearl Beach and a typical beach profile measured on 23 October 1996. The beach profile was measured across a cusp embayment, approximately 300 m north of the southern end of the Pearl Beach (at $y = 300$ m in Figure 5)

FIELD SITE AND METHODS

Pearl Beach is a 1 km long embayed beach located on the northwestern shore of Broken Bay, 50 km north of Sydney, Australia (Figure 2). The shoreline has a typical parabolic or zeta form, with the radius of curvature increasing northwards (Figure 3A). The wave energy level along Pearl Beach increases gradually from the sheltered southern end of the embayment to the more exposed northern end. As a result, runup height, berm height and mean grain size increase in the northward direction (Bryant, 1979; Hughes and Cowell, 1987). Pearl Beach is perennially a reflective beach system and well developed cusps (Figure 3B) are typically present along the whole embayment (Cowell, 1982).

The deep-water wave climate of Broken Bay is dominated by a persistent southeasterly swell, superimposed on a highly variable wind wave regime (Short and Wright, 1981). The modal deep-water wave in the region has a significant height of 1·5 m, a period 10 s, and is incident from the southeast (Lawson and Abernathy, 1975). Pearl Beach experiences semi-diurnal tides with a mean tidal range of 1·2 m (Department of Defence, 1996).

Morphological measurements were conducted during the morning of 23 October 1996 when pronounced beach cusp morphology was present along the Pearl Beach embayment. The beach cusps had a ‘fresh’ appearance: the cusp features were sharp and the sand surface was relatively undisturbed by footprints. Considering the popularity of Pearl Beach, this suggests that the beach cusps were recently formed. Figure 4 shows the offshore significant wave height and period over the second half of October 1996. The data were obtained from a wave rider buoy deployed in 100 m water depth, 25 km southeast of Pearl Beach. A major storm occurred on 20 and 21 October 1996, with the offshore wave height exceeding 3 m and the wave period less than 8 s. A storm of such intensity can be expected to cause significant erosion on Pearl Beach (Hughes and Cowell, 1987). Modal and relatively constant wave conditions were experienced for several days following the passage of the storm, including the day of the

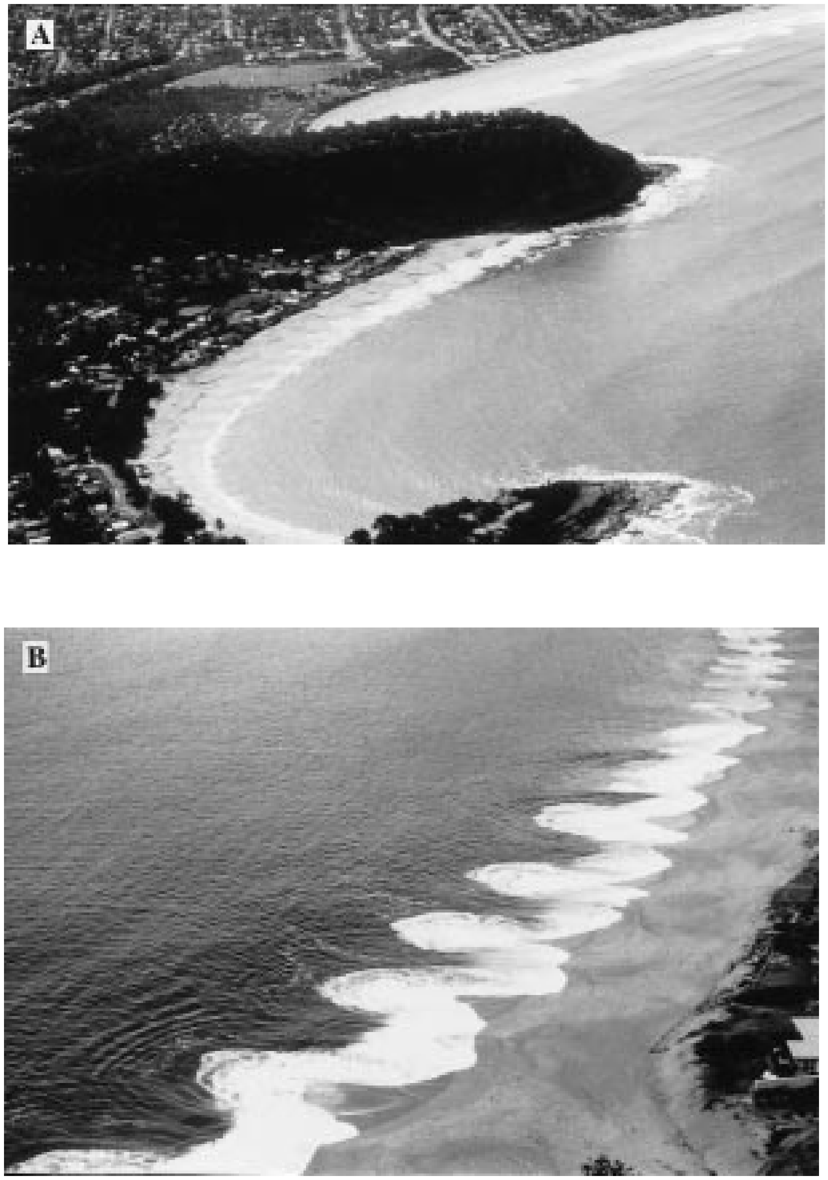

Figure 3. (A) View of Pearl Beach (the beach at the foreground) taken from the southwest indicating zeta-shaped configuration of the embayment (the beach at the background is Umina Beach; cf. Figure 2). (B) Well-defined cusp morphology and three-dimensional swash circulation present at the northern end of Pearl Beach (photos: P. J. Cowell)

morphological survey. The field observations and wave data imply that the beach cusp morphology observed on 23 October 1996 formed on the preceding day and/or night (22 October 1996) under the influence of post-storm, modal wave conditions.

Forty-five beach cusps were present on Pearl Beach on the day of the survey, suggesting an average cusp spacing of 22–23 m. A typical beach profile is shown in Figure 2, indicating a relatively steep beachface, a low-tide beach step and a lower gradient nearshore. Mini-profiles were surveyed across

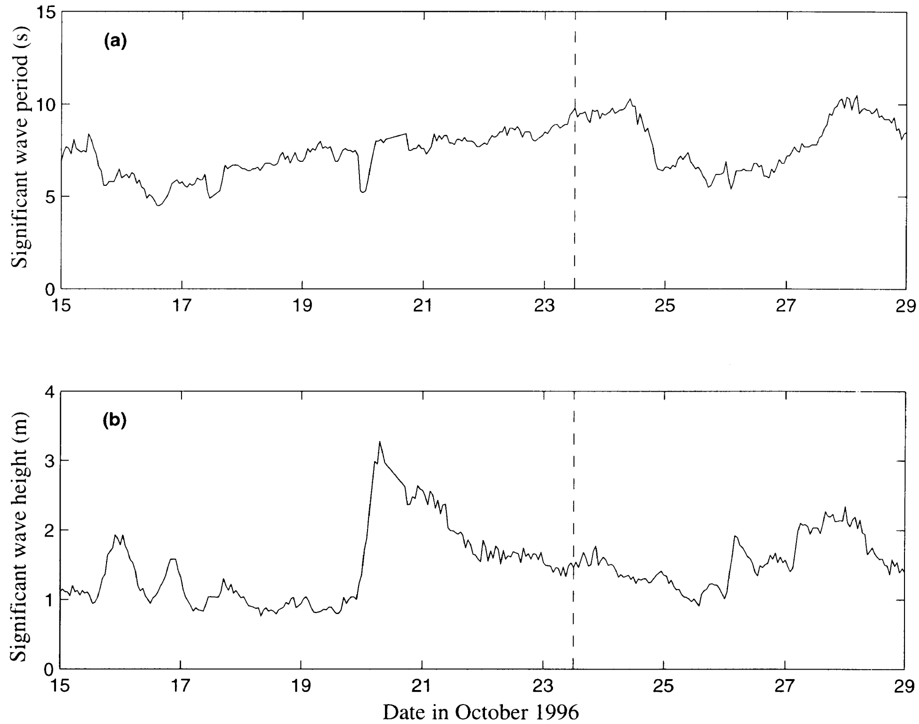

Figure 4. Offshore wave conditions from 15–29 October 1996: (a) significant wave period; (b) significant wave height. Dashed line indicates the time of the survey

every cusp horn and embayment using an electronic survey station positioned in the centre of the embayment. Along each profile, three points were surveyed: (1) the berm crest; (2) the mid-beachface position; and (3) the downrush limit (the lowest point on the beachface that falls dry during swash action at the time of the survey). The measured elevations and positions were related to the Australian Height Datum (AHD; approximate mean sea level) and the Australian Map Grid coordinates (AMG), respectively.

Four parameters were derived from the data set: (1) cusp spacing λ; (2) beachface gradient tan β; (3) cusp prominence ε; and (4) horizontal swash excursion S. These parameters were determined for each individual beach cusp on the basis of two mini-profiles across the cusp horns and one mini-profile across the embayment. The cusp spacing was determined from the alongshore distance between two cusp horns. The beachface gradient was computed for each mini-profile from the berm crest to the downrush limit. The average beachface gradient was then obtained by averaging the gradient of the cusp horn (mean of the two horn gradients) and embayment. The prominence of the beach cusp morphology was computed using Equation 4. Owing to the stochastic nature of natural waves, the horizontal swash excursion is necessarily a statistical parameter. Therefore, monitoring of the swash motion over a significant amount of time (at least 30 min) is required before the appropriate swash length parameter (e.g. mean, significant and maximum swash excursion) can be computed. A more pragmatic definition of the swash excursion was used here for obvious reasons. The swash excursion was defined as the horizontal distance between the berm crest (high-tide runup limit) and the high-tide downrush limit, where high tide refers to the tidal cycles during which the beach cusps developed. The high-tide

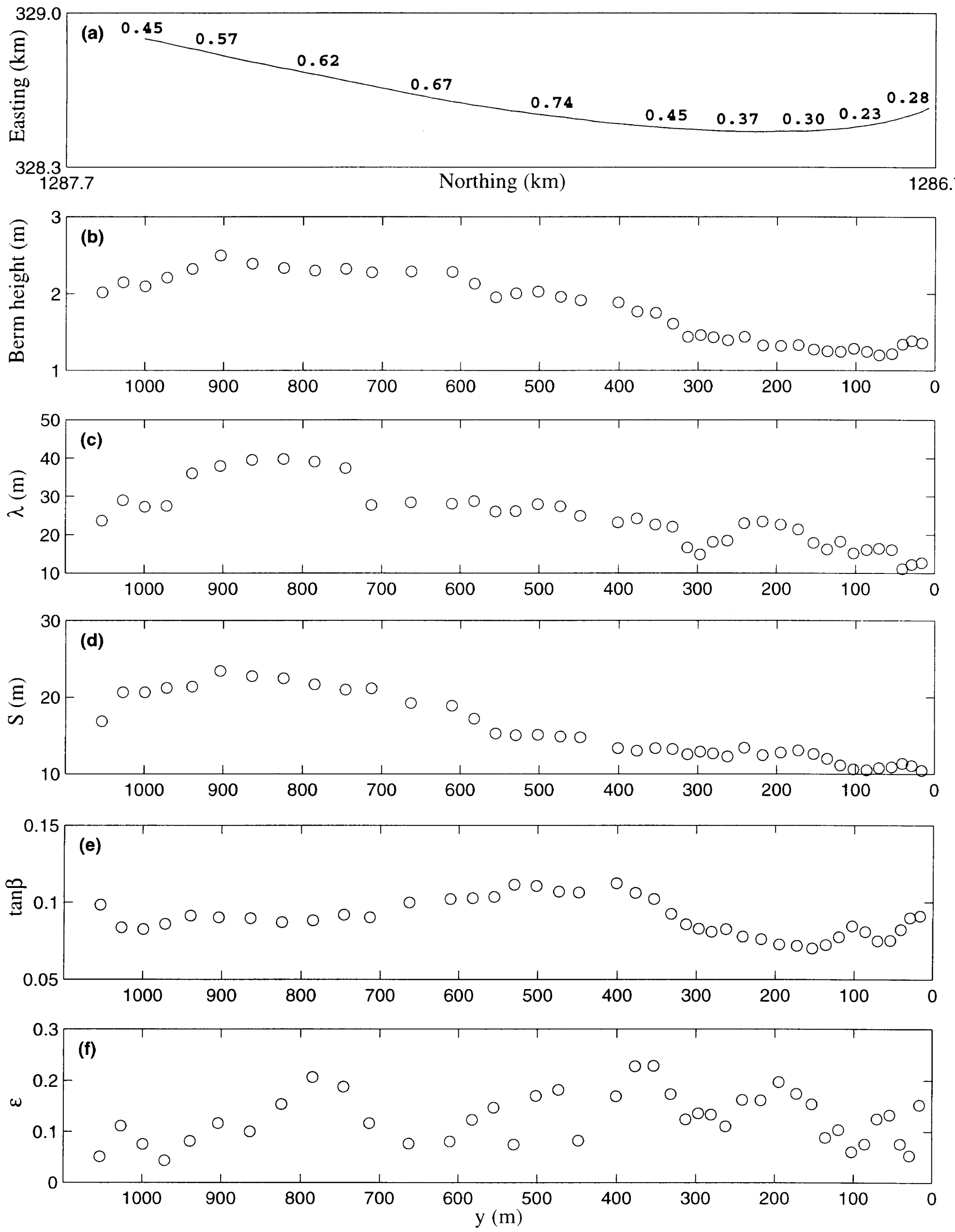

Figure 5. (a) Coastline of Pearl Beach with median sediment size (in mm) and alongshore variation in: (b) berm crest elevation relative to AHD; (c) cusp spacing λ; (d) swash excursion S; (e) beachface gradient tan β; (e) cusp prominence ε

downrush limit was taken as 0·2 m below high-tide level on the basis of the observation that the surveyed downrush limits were all between 0 and 0·4 m below the water level at the time of the survey. The swash excursion was then computed using $S = [BH-(HT-0{\cdot}2)]/\tan\beta$, where BH represents the berm height (in m AHD) and HT is high-tide level during cusp formation (0·6 m AHD).

A sediment sample was obtained from the mid-beachface position of every fifth cusp horn and analysed using a settling tube. The equations of Hallermeier (1981) were used to convert the median settling velocity to sediment size.

RESULTS

The morphological characteristics of 45 beach cusps were determined, but four cusps were discarded from further analysis because of obvious measurement errors. The wave conditions during the cusp formation (and also during the survey) were characterized by an offshore significant wave height of around 1·5 m and a significant wave period of 8–9 s (spectral peak period was 1–2 s longer). Refraction/diffraction of the incident waves resulted in significant attenuation of the wave height. Visual field observations indicated a breaker height of around 0·25 m at the most protected southern end of the embayment, increasing to approximately 1·25 m at the least sheltered northern end. Along the entire embayment the incident waves approached the coast with their crests parallel to the shoreline and significant longshore currents were not observed.

Pearl Beach exhibits a curved section at the southern end of the embayment and a straight section at the northern end (Figure 5a). The coarsest sediments (0·7 mm) were found in the centre of the embayment while the finest sediments (0·3 mm) occurred at the southern end (Figure 5a). Towards the north, a gradual increase in the elevation of the berm crest (from 1 to 2·5 m), cusp spacing (from 10 to 40 m) and swash excursion (from 10 to 25 m) is apparent (Figure 5b–d). This northward trend is attributed to an increase in the wave energy level. The beachface gradient was closely related to the sediment size (Figure 5e). Flattest slopes were found at the southern end of the embayment ($\tan\beta = 0{\cdot}07$), steepest slopes occurred in the centre ($\tan\beta = 0{\cdot}11$), and intermediate slopes were found at the northern end ($\tan\beta = 0{\cdot}09$). The prominence of the beach cusps was variable ($\varepsilon = 0{\cdot}05$–$0{\cdot}025$) and did not display any obvious pattern (Figure 5f).

Equations 1 and 2 were used to determine the alongshore variation in the incident wave period required to produce the observed trend in cusp spacing according to the edge wave model. The required period ranged from 7 to 13 s for the sub-harmonic case and from 10 to 17 s for the synchronous case (Figure 6a). If the surveyed beach cusps were all formed during one event (as is assumed), the incident wave period must have been constant along the beach. To estimate this constant period, cusp spacing was plotted against beach gradient (Figure 7a) and least-squares analysis between these variables was conducted. To explain the observed spacing of the cusps using edge wave theory, the line of best fit through the data points should have a slope of gT^2/π for the sub-harmonic case and $gT^2/2\pi$ for the synchronous case. However, no significant relation (at the 95 per cent significance level) could be demonstrated between cusp spacing and beach gradient.

The alongshore variation in the ratio of cusp spacing to swash excursion was investigated to validate the self-emergence model of beach cusp formation. Individual λ/S ratios ranged from 1 to 2, but did not display any longshore trend (Figure 6b). Linear least-squares analysis of Equation 3 yielded $f = 1{\cdot}57$ with a coefficient of determination (r^2) of 0·82 (Figure 7b).

The parameter $\varepsilon(S/\lambda)^2$ did not exhibit an alongshore trend and ranged from 0·025 to 0·1 (Figure 6c). Linear least-squares analysis was used to determine the value of coefficient K in Equation 5 and $K = 4{\cdot}46$ was obtained with a coefficient of determination (r^2) of 0·73 (Figure 7c). Rearranging Equation 5 and inserting $K = 4{\cdot}46$ results in $\varepsilon = 0{\cdot}05(\lambda/S)^2$. A scatter diagram of λ/S versus ε further suggests that large values of λ/S ($>1{\cdot}5$) are associated with pronounced cusps ($\varepsilon > 0{\cdot}1$), whereas small values of λ/S ($<1{\cdot}5$) are related to subdued cusps ($\varepsilon < 0{\cdot}1$) (Figure 8).

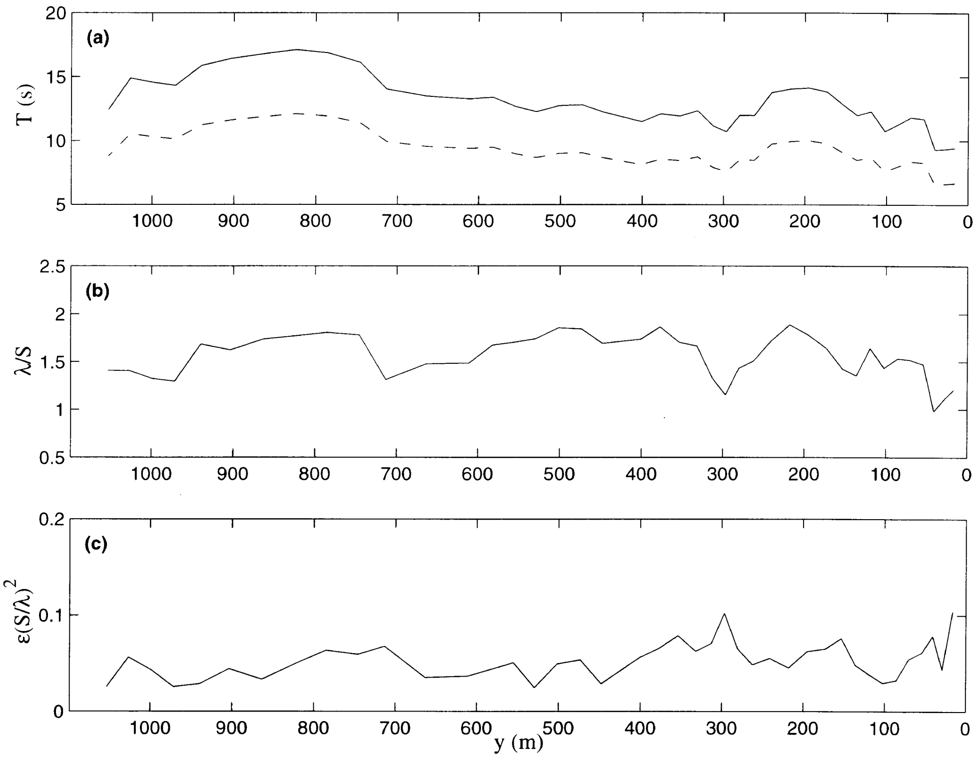

Figure 6. Alongshore variation in: (a) incident wave period required to produce the observed cusp spacing according to the edge wave model for sub-harmonic (dashed line) and the synchronous case (solid line); (b)λ/S; (c) $\varepsilon(S/\lambda)^2$

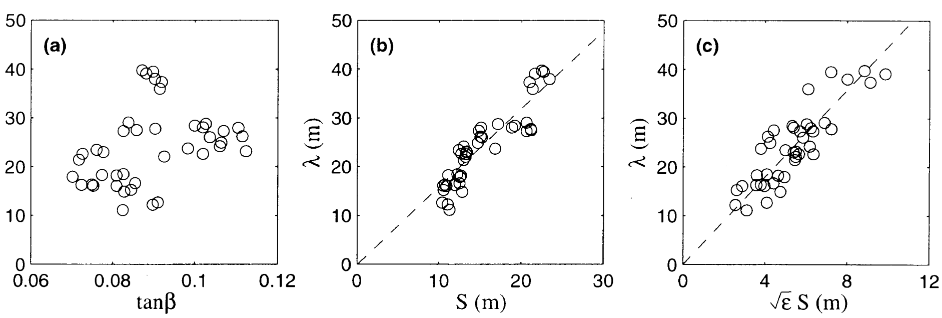

Figure 7. Scatter plots of: (a) λ versus tanβ; (b) λ versus S; (c) λ versus $\sqrt{\varepsilon}S$. The dashed lines in (b) and (c) represent linear least-squares lines fitted to the data (forced through the origin)

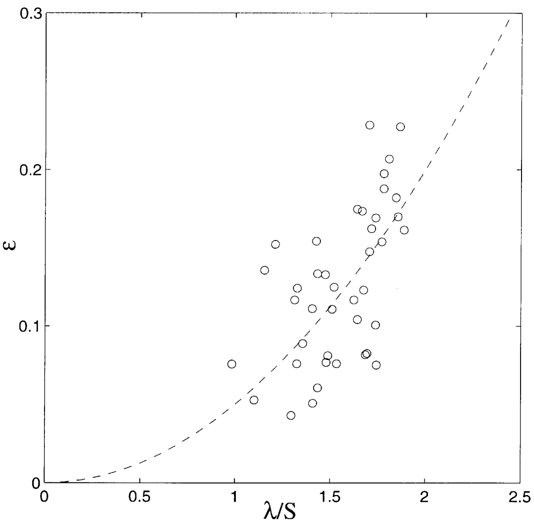

Figure 8. Scatter plots of ε versus λ/S. The dashed line represents $\varepsilon = 0{\cdot}05(\lambda/S)^2$

DISCUSSION

Analysis of the morphological characteristics of 41 beach cusps present along the shoreline of an embayed beach revealed a highly significant relation between cusp spacing and swash excursion, providing strong support for the self-organization model of beach cusp formation. Least-squares analysis yielded $\lambda = 1{\cdot}57S$, which is nearly identical to the model results obtained by Dean and Maurmeyer (1980) and the field results of Takeda and Sunamura (1983), and is within the range quoted by Werner and Fink (1993). A relatively wide range of individual λ/S ratios was represented in the data ($\lambda/S = 1$–2), and further analysis indicated that those cusps with the largest λ/S ratios were characterized by the most pronounced morphology (largest ε values), and vice versa. A similar trend was present in the field data reported by Dean and Maurmeyer (1980; their Tables 2 and 3) which indicated subdued cusps ($\varepsilon = 0{\cdot}1$) with $\lambda/S = 1{\cdot}9$, and pronounced cusps ($\varepsilon = 0{\cdot}25$) with $\lambda/S = 3{\cdot}2$. Beach cusp morphology is maintained by a three-dimensional swash flow circulation pattern characterized by deflection of the wave uprush from the horns into the adjacent embayments. This pattern is referred to as horn-divergent flow and the extent of horn divergence decreases with λ/S and increases with ε (Masselink and Pattiaratchi, 1998). The inverse relationship between λ/S and ε supports the notion that equilibrium conditions on beach cusp morphology can be defined in terms of the extent of horn-divergent swash flow; beach cusps with small λ/S ratios need to build up steeper horns and flatter embayments (large ε) to attain the same amount of horn divergence required for equilibrium than less pronounced beach susps (small ε) with large λ/S ratios. The commencement of this equilibrium state represents the switch from positive to negative feedback processes in the self-organization model of Werner and Fink (1993). The present data suggest that the equilibrium condition is quantified by $\varepsilon(\lambda/S)^2 = 0{\cdot}05$, in good agreement with the numerical and field results of Masselink and Pattiaratchi (1998).

This investigation and its conclusions are based on a number of assumptions that require justification. These assumptions are: (1) the observed cusp morphology was formed on the previous day/night; (2) the

edge wave dispersion relationships given by Equations 1 and 2 are valid; (3) the measured beach gradient is representative for the beachface slope during cusp formation; and (4) the swash excursion during the formation of the cusps is well parameterized by the distance from berm crest to downrush limit.

(1) The beach cusps that were present on Pearl Beach on 23 October 1996 are considered to have formed the preceding day and/or night. The arguments to support this are the fresh and undisturbed appearance of the cusps during the survey, and the occurrence of a severe storm event several days prior to the field investigation. It is assumed that the cusp morphology developed during the post-storm, beach recovery phase under accretionary waves, conditions which are conducive to beach cusp formation (e.g. Takeda and Sunamura 1982; Masselink *et al.,* 1997). However, the possibility that the configuration (i.e. alongshore variation in cusp spacing) of the beach cusps measured during the field survey was inherited from pre-storm morphology cannot be fully refuted. Masselink *et al.* (1997) document the formation of beach cusps following a storm at the same location and with the same spacing as cusps present prior to the storm, and argue that the remains of the pre-storm morphology formed the template for the new beach cusps.

(2) Strictly speaking, the edge wave dispersion equations given by Equations 1 and 2 are inappropriate because they represent the idealized condition of a planar beach (an additional limitation is that longshore currents must be absent, but this condition is relatively easily met on embayed beaches subject to low energy swell conditions, such as Pearl Beach). Natural beaches are generally non-planar and in these cases the relation between edge wave period and wave lengths needs to be determined numerically (Holman and Bowen, 1979; Holland and Holman, 1996). However, the purpose of the present paper is not to compare the observed spacing of individual cusps with that predicted using edge wave theory, but to investigate whether edge wave theory can explain the observed alongshore trend in cusp spacing. For both planar (Equations 1 and 2) and non-planar beaches (numerical solution), edge wave theory predicts that for a constant wave period, the beach cusp spacing increases with increasing beach gradient. The data clearly indicate that this is not the case. On the contrary, in the middle section of Pearl Beach (from $y = 400$ to 850 m), the cusp spacing increases while the beach gradient decreases (Figure 5). The edge wave model can only be made to 'fit' the morphological data by invoking several episodes of beach cusp formation, each triggered by a different incident wave period. This is not supported by the observed wave period during cusp formation which remained at approximately 9 s (Figure 4). Even if the beach cusp configuration was inherited from pre-storm morphology (cf. previous paragraph), it would still be very difficult to explain these opposite alongshore trends in cusp spacing and beach gradient using edge wave theory.

(3) It may be suggested that the measured beach gradients were not representative for the beachface morphology during beach cusp formation. Two questions need to be answered. First, does the mean of the cusp horn and embayment gradient represent the beachface gradient during cusp formation? Detailed morphological measurements reported by Masselink *et al.* (1977) indicate that cusp horns result from a local steepening of the beachface gradient by sediment accretion, whereas sediment erosion and flattening of the beach slope result in the formation of cusp embayments. Hence, the mean of the horn and embayment gradient is characteristic of the pre-cusp beachface gradient. Second, is it possible that the beachface gradient has been modified by swash processes following the formation of the cusps? The alongshore trend in beach gradient observed on Pearl Beach is similar to the trend in sediment size (Figure 5), conforming to the well established correlation between grain size and beach gradient (Bascom, 1951). It is likely that some modification in the beach gradient has occurred following cusp formation. However, the longshore trend in beach gradient measured during the field survey is typical for Pearl Beach and can be considered representative for the conditions during cusp formation.

(4) The present investigation considers that the swash excursion during the formation of the cusps is well estimated by the horizontal distance between the berm crest and the high-tide downrush limit during cusp formation. Such a definition of swash excursion is rather 'robust' and insensitive to changing hydrodynamic conditions; the berm crest is the morphological expression of the runup limit and the

downrush limit is more related to the tidal water level than the incident wave conditions (Nielsen and Hanslow, 1991). However, such errors only affect the actual λ/S ratios, and hence the f value, not the strength of the relationship between swash length and cusp spacing.

SUMMARY

The variation in cusp morphological characteristics was investigated along a 1 km long embayed beach to test the edge wave model and the self-organization mechanism of beach cusp formation. Application of edge wave theory to the observed beach cusp spacing predicted an incident wave period during cusp formation of 7–13 s for the sub-harmonic edge wave case (Equation 1) and 10–17 s for the synchronous case (Equation 2). During cusp formation, the wave period was approximately 9 s, which is within the range predicted by the sub-harmonic edge wave case. However, the alongshore trend in cusp spacing was clearly at odds with the edge wave model of cusp formation. Edge wave theory states that cusp spacing increases with beach gradient; however, no statistically significant relationship could be demonstrated between these two variables. In contrast, a strong relationship was found between cusp spacing and swash excursion, with the largest cusps associated with the longest swash length, and the smallest cusps with the shortest swash length. Linear least-squares analysis yielded $\lambda = 1{\cdot}57S$, providing support for the self-organization theory of beach cusp formation. According to the self-organization mechanism, beach cusp morphology evolves as a result of positive feedback processes between horn-divergent swash flow and three-dimensional beachface morphology. At some stage during cusp development an equilibrium condition is expected to be attained whereby the swash circulation is such that no further net erosion or accretion occurs. For the beach cusps on Pearl Beach this condition may be parameterized by $\varepsilon(S/\lambda)^2 = 0{\cdot}05$.

ACKNOWLEDGEMENTS

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